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PAIR EXPLOSION IN AN EXPONENTIAL ATMOSPHERE

V. A. Andrushchenko

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The problem of the interaction of two coaxial explosions in a barometric atmosphere is solved numerically based on the complete system of Navier-Stokes equations. Basic regularities that occur in the interference of two spherical shock waves of different intensities are studied. The last stage of the processes, when shock wave processes become unimportant and convection plays a dominant part, is investigated.

The class of phenomena involving a collision of spherical shock waves is extremely wide, and the scale of such phenomena also ranges widely, from laser sparks and electric discharges [1, 2] to objects of astronomical dimensions [3]. Generally, the interaction of two spherical shock waves (SW) is a three-dimensional nonstationary problem, and its solution is beyond the potentialities of modern computers so far (in a full-scale experiment, this problem was treated in [4]). In this connection it is reasonable to turn to simpler formulations; in particular, we may examine the problem of a pair explosion in an exponential atmosphere when explosion centers lie on a common vertical.

In the present work we present some results obtained in a numerical calculation that reflect basic regularities that occur in the interference of two spherical SW of different intensities. We investigated the following aspects of a pair explosion in an exponential atmosphere: conversion of a regular SW reflection to a Mach reflection, SW propagation over central regions with pronounced entropy nonuniformities, and the appearance and merging of complex vortex stuctures in the development of convective flows.

1. As a mathematical model of the phenomenon, we take the complete system of nonstationary Navier-Stokes equations for a compressible heat-conducting gas in cylindrical coordinates (r, z) and the equation of state for an ideal gas (see [5]).

The problem is solved in the rectangular region $V(t) = \{0 \le r \le f(t), \varphi_-(t) \le z \le \varphi_+(t)\}$ with moving right-hand, upper, and lower boundaries that are located in a practically a undisturbed medium and shift as the SW expands.

The boundary conditions are the following:

$$\text{for} \quad r = 0 \quad u = \partial v/\partial r = \partial p/\partial r = \partial T/\partial r = 0\,;$$

$$\text{for} \quad \begin{matrix} r = f(t) \\ z = \varphi_\pm(t) \end{matrix} \quad u = v = 0\,, \quad p = p_a(z)\,, \quad T = T_0$$

(here, $p_a(z)$ is the pressure in an exponential atmosphere at the altitude z and T_0 is the temperature). The initial conditions are as follows: inside two regions involved in explosions, the parameter distributions correspond to solutions of the explosion problem that are obtained using one-dimensional methods (see [5]), and outside them, they correspond to parameters of an undisturbed barometric atmosphere. Dissipative factors, namely, viscosity and radiative heat conduction, are taken into account: the coefficients μ and k are approximated by power-law relations $\mu \sim T^\omega$ and $k \sim T^\alpha \rho^\beta$ (see [6]).

The initial equations as well as the boundary and initial conditions are made dimensionless with the aid of the following characteristic parameters: the altitude of the homogeneous atmosphere Δ is the linear scale, $\sqrt{\Delta/g}$ and $\sqrt{\Delta g}$ are the time and velocity scales, and $p_0 = p_a(z_1')$ and T_0 are the pressure and temperature scales. Thus, the governing parameters of the problem are:

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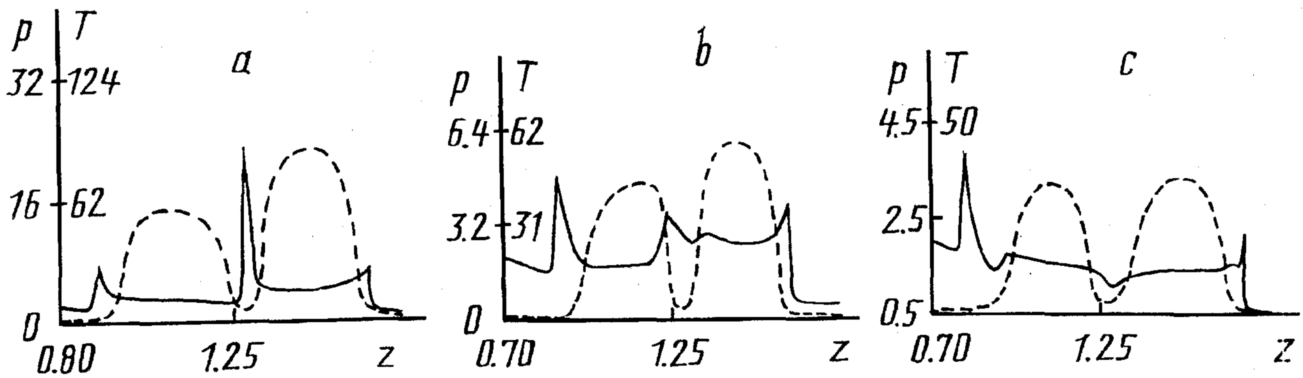

Fig. 1. Pressure and temperature distributions along the symmetry axis z: a) $t = 0.008$; b) 0.025; c) 0.07.

$$z_1 = z_1'/\Delta, \quad z_2 = z_2'/\Delta, \quad R_1 = R_1'/\Delta, \quad R_2 = R_2'/\Delta,$$

$$\mathrm{M} = (\Delta g/\gamma R^0 T_0)^{1/2}, \quad \mathrm{Re} = \Delta\sqrt{\Delta g}\,\rho_0/\mu_0,$$

$$\mathrm{Pr} = c_p \mu_0 \rho_0/k_0, \quad \gamma = c_p/c_V, \quad \alpha, \ \beta, \ \omega.$$

2. The initial system of differential equations is discretized using an implicit difference scheme of splitting with respect to coordinate directions and functions [5]. To suppress nonphysical oscillations resulting from nonmonotonicity of the chosen scheme, regularization of the numerical solution is used. Calculations are performed on a uniform 71×251 grid in the r and z directions, respectively.

3. We now proceed to a consideration of the results of calculating a pair explosion with the following values of the governing parameters:

$$\mathrm{Re} = 10^7; \quad \mathrm{Pr} = 1; \quad \mathrm{M} = 0.6; \quad \gamma = 1.4; \quad \omega = 1.5;$$

$$\alpha = 1.5; \quad \beta = -2; \quad R_1 = 0.2; \quad R_2 = 0.15.$$

The pressure drop on the front of the lower SW at $t = 0$ is $p_1 = 7$, and of the upper SW, $p_2 = 20$. (Clearly, with the above Reynolds numbers, dissipative factors manifest themselves only in the central regions of the explosions, where the temperatures are high and the densities are low.)

The evolution of the interference pattern of the shock waves may be seen in Fig. 1, in which distributions of the pressure p (solid curves) and temperature T (dashed curves) along the symmetry axis z are presented for three moments of time ($t = 0.008$, 0.025, and 0.07).

Figure 1a illustrates the instant immediately after the collision has begun – the pressure intensity at the contact rises sharply. The interaction occurs in a dense gas, in a zone with a relatively low temperature; as a result, two SW are formed that move in opposite directions.

Figure 1b shows the next stage of the process, where the SW interact with the boundaries of a sharp density change at the entrance to the hot central regions of the explosions. Here, these waves break up into SW that continue to move in the previous direction and rarefaction waves that are directed toward the zone of the initial contact. Once the SW fronts enter hot rarefied gas, their propagation velocities increase appreciably, the intensities fall, and the shocks themselves are strongly "smeared." This effect is known both from laboratory experiments on the interaction of laser sparks [1] and from data of calculation of the collision of SW with thermals [7]; it occurs when the velocity of the front prior to penetration into hot gas is smaller than the local velocity of sound in it. The more heated the layers that the fronts enter, the more strongly are they accelerated and smeared; in turn, intense SW markedly "deform" the hot central zones (see the distribution of T in Fig. 1b).

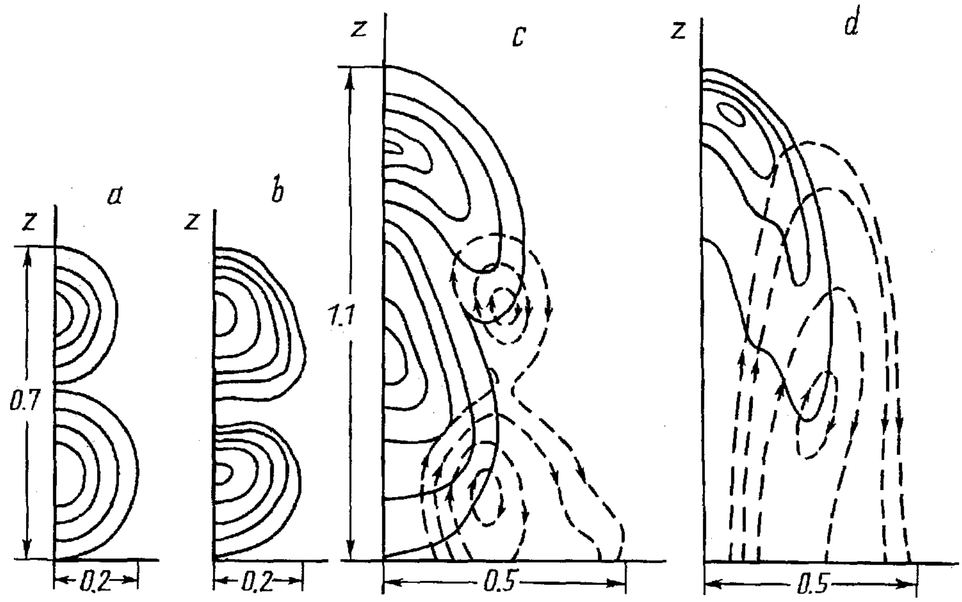

Fig. 2. Isotherms (solid curves) and streamlines (dashed curves) for four instants of time: a) $t = 0$; b) 0.07; c) 0.47; d) 1.06.

Upon escaping from rarefied regions, the shocks interact with the boundaries of a sharp density rise, breaking up into SW pairs that move in opposite directions: some are directed backward into the hot zones and the others follow the leading fronts (see Fig. 1c). After a time, the latter overtake the leading SW, increasing their intensity to some extent. This overtaking is realized if the leading shocks are still rather intense by this time (as is in the version considered). Otherwise, zones of return flow with negative phases of excess pressure $\Delta p = p - 1$ and velocity are formed behind the primary SW with small pressure drops, and then situations are possible where the secondary shocks do not overtake the leading shocks (see [8]). This occurs when the total negative momentum of the return motion is greater than the corresponding positive momentum of the overtaking wave.

At the initial stage of the interference of the SW, when angles between the fronts are small, the reflection is of regular character. After a limiting angle has been passed, the interaction of the SW becomes a March interaction. By the time $t = 0.07$, a complex transitional structure in the form of a suspended compression shock is formed between the fronts; it is precisely here that maximum pressures are realized (see [2] for details).

Calculations demonstrated that the shock-wave phase of the interaction of two explosions, when the primary and secondary shock waves exert a decisive effect on the distribution of gasdynamic parameters in the disturbed region, is relatively short, lasting for up to about $t = 0.07$. After that, the secondary SW weaken so much that they do not play any noticeable role in subsequent evolution of the gas volume involved in the motion. Thus, by the time $t = 0.11$ the secondary shocks, formed upon the escape of the SW from the hot explosion regions, arrive at the rarefied zone near the initial contact. But a head-on collision of these shocks leads, because of their low intensity, only to an insignificant increase in the pressure in the region of the initial contact.

We now pass on to a consideration of the final, convective, stage of interaction. Figure 2 gives distributions of isotherms (solid curves) for four instants of time. In Fig. 2a, isotherms in across section at $t = 0$ (prior to commencement of the collision) have the form of isolated concentric semicircles. Subsequently, as a result of the collisional interaction, they bend markedly, assuming shapes in the directions of SW propagation that characterize a single thermal that ascends in an atmosphere under the action of gravity, but for a significantly later time (see Fig. 2b, $t = 0.07$). Such collisional deformation of thermal elements was noted in experiments in shock tubes (see, for example, [9]) and was also obtained in calculations [7].

Then, as the convection intensifies and the wave processes fade out, the flow develops similarly to an ascend of two coaxial thermals (see [10]). The upper hot region ascends by the law of isolated formation $z \sim t^{1/2}$, and the lower region, floating up in the wake of the upper one with insignificant resistance of the ambient medium, moves following the law $z \sim t^2$ (the altitudes of ascend are specified by the points with maximum temperatures). Here, the upper hot cloud takes the form of a "mushroom cap," and the lower, of the "stalk" (see Fig. 2c, $t = 0.47$). Gradually the lower thermal overtakes the upper one (as if the "stalk" is drawn into the "cap") and merges with it (see Fig. 2d, $t = 1.06$). The resulting monoformation floats up by the law of a single thermal $z \sim t^{1/2}$ and has the characteristic form of a "mushroom cap," with a temperature maximum that has already departed from the symmetry axis (see Fig. 2d).

Early in the development of the convective motion a complex vortex structure is generated in the form of a pair of vortex tories with a unidirectional, namely, clockwise, gas rotation (see the dashed curves in Fig. 2c; here streamlines constructed based on the instantaneous velocity vector field are presented). As the thermals merge, the vortex field also transforms, and a toroidal monovortex is formed by the time $t = 1.06$ (see the dashed curves in Fig. 2d). It should be noted that a monovortex is formed somewhat after the temperature structures merge into a unified formation with a single maximum.

Generally, the process of interaction of two explosions may be divided into three stages. The first stage ($t < 0.07$) is a shock wave one, it is characterized by an intricate pattern of SW interference and includes regular and Mach reflections in a head-on collision of the leading fronts and interactions of secondary SW with one another and with zones of entropy nonuniformities. The second is a shock-convective stage ($0.07 \lesssim t \lesssim 0.14-0.18$). At this stage, on the one hand, the influence of secondary shocks on flow development is still appreciable and, on the other hand, intense convective flows already arise due to gravity. And lastly comes the final, third, stage ($t > 0.18$) in which convection plays an absolutely dominant role and wave disturbances are insignificant.

NOTATION

t, time; r, z, cylindrical coordinates; $\mathbf{v} = (u, v)$, velocity; ρ, density; p, pressure; T, temperature; μ, k, dynamic viscosity and thermal conductivity; $V(t)$, calculation region; $f(t)$, $\varphi_{\pm}(t)$, boundaries of the calculation region; z_1', z_2', altitudes of the centers of the lower and upper explosions; R_1', R_2', initial radii of the regions involved in the explosions; Δ, altitude of the homogeneous atmosphere; g, acceleration due to gravity; γ, adiabatic exponent; α, β, ω, parameters; M, Mach number; Re, Reynolds number; Pr, Prandtl number; c_p, c_V, specific heats.

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